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Predictive Functional Control Based on Fuzzy Model: Comparison with Linear Predictive Functional Control and PID Control

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Abstract. The implementation of the fuzzy predictive functional control (FPFC) on the magnetic suspension system is presented in the paper. The magnetic suspension system was in our case the pilot plant for magnetic bearing and is an open-loop unstable process, therefore a lead compensator was used to stabilize it. The high quality control requirements were a-periodical step response and zero steady-state error. Adding the integrator to a feedback causes overshoot. The solution to the problem was cascade control with fuzzy predictive functional controller in the outer loop. To cope with the unknown model parameters and the nonlinear nature of the magnetic system, a fuzzy identification based on FNARX model was used. After successful validation the obtained fuzzy model was used for controller design. The FPFC is compared with a cascade linear predictive functional control (PFC) and PID control. The results we obtained with the FPFC are very promising and hardly comparable with conventional control techniques.

Key words: fuzzy identification, predictive control, real-time control.

1. Introduction

Predictive control is the name for several different control methods such as: Generalized Predictive Control (GPC) (Clarke *et al.*, 1987), Dynamics Matrix Control (DMC) (Cutler and Ramaker, 1980) and Predictive Functional Control (PFC) (Richalet *et al.*, 1978). The control law is based on the prediction, obtained with the model of the controlled process. Control action is calculated in the way to minimize the difference between the predicted process output and the reference signal over a certain time horizon. Predictive controllers generally exhibit remarkable robustness with the respect to the model mismatch and unmodelled dynamics (Camacho and Bordons, 1995). Very good results were also achieved in combination with time delay processes (Camacho and Bordons, 1995). When based on fuzzy model, predictive controllers proved to be very convenient for strongly nonlinear processes (Škrjanc and Matko, 2000, 2001).

In this paper, a method of predictive functional control is applied to a nonlinear process, namely a stabilized magnetic suspension. The researches, which have been made, are relating to the idea of the magnetic bearing. The high-quality control requirements (short settling time with a-periodical step response and without steady-state error) cannot be achieved using PID controller. For that reason nonlinear Fuzzy PFC (FPFC) was applied. The real-time experiment of controlling the stabilized magnetic suspension was made, to gain a deeper insight into process behaviour. Due to insufficient knowledge of the process, the identification, based on the FNARX model, has been made. The prediction was based on global linear model (Škrjanc and Matko, 2000, 2001), written in the form of Takagi–Sugeno type of the fuzzy model (Škrjanc and Matko, 2000; Takagi and Sugeno, 1985; Sugeno *et al.*, 1991). To calculate the H step-ahead output prediction of the third-order model with complex poles in the transfer function, the model had to be written in the state space domain (Škrjanc and Matko, 2001).

The paper is organized as follows: The magnetic suspension system is presented in Section 2. Section 3 describes the concept of fuzzy identification. Predictive functional control principles and design are given in Section 4 and the real-time implementation of control algorithm together with the comparative analysis with the linear PFC and PID control on the magnetic suspension system is presented in Section 5.

2. Magnetic Suspension System

Magnetic suspension system (Manual for MA401 Magnetic Suspension System) consists of an electromagnet, a coil and a distance sensor. Its basic principle is shown in Figure 1, where u_{RL} and i stand for voltage and current of the electromagnet respectively, R and L are resistance and inductance of the electromagnet, c is unknown parameter, m is the mass of the coil and l is the distance between the electromagnet and the coil. The goal is to control the distance l by the control variable u_{RL}.

Using the second Newton law, we can write

$$mg - F_m = m\frac{\mathrm{d}^2 l}{\mathrm{d}t^2}. \tag{1}$$

The magnetic force depends on the current i, the distance l and the parameter c.

$$F_m = c\frac{i^2}{l^2}. \tag{2}$$

The electrical part of the system is modelled with the following equation

$$u_{RL}(t) = L\frac{\mathrm{d}i}{\mathrm{d}t} + Ri. \tag{3}$$

The sensor changes the distance to voltage. The function is linear with the offset.

$$y_p = K_{\mathrm{sens}} l + U_{\mathrm{sens}}, \quad K_{\mathrm{sens}} = -4\ \mathrm{V/mm},\ U_{\mathrm{sens}} = 10\ \mathrm{V}. \tag{4}$$

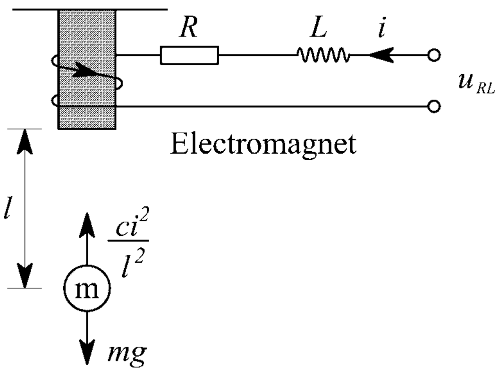

Figure 1. Basic principle of magnetic suspension.

Between the controller (PC) and the electromagnet is interface (amplifier) with the following function:

$$u = K_{\text{act}} u_{RL} + U_{\text{act}}, \qquad K_{\text{act}} = 2,\ U_{\text{act}} = -10\ \text{V}. \tag{5}$$

Combining Equations (1), (2) and (3) the nonlinear unstable differential equation is obtained

$$\frac{Lm}{c}\left(g - \frac{\mathrm{d}^2 l}{\mathrm{d}t^2}\right)\frac{\mathrm{d}l}{\mathrm{d}t} - \frac{Lm}{2c} l \frac{\mathrm{d}^3 l}{\mathrm{d}t^3} + \frac{Rm}{c} l\left(g - \frac{\mathrm{d}^2 l}{\mathrm{d}t^2}\right) - u_{RL}\sqrt{\frac{m}{c}\left(g - \frac{\mathrm{d}^2 l}{\mathrm{d}t^2}\right)} = 0. \tag{6}$$

In spite of our very poor knowledge about the magnetic system parameters, we managed to stabilize it. The following lead compensator was used:

$$G_{cc}(s) = 4.5\left(\frac{s + 40}{s + 400}\right), \tag{7}$$

which can be in discrete domain at 2 ms sampling time expressed as:

$$G_{cd}(z^{-1}) = 4.5\left(\frac{1 - 0.9449 z^{-1}}{1 - 0.4493 z^{-1}}\right). \tag{8}$$

Also the feed-forward compensation of gravity of 2 V was applied.

Robustness of the lead compensator assures the stability in the whole operating range. It is well known that the steady-state error cannot be eliminated by just using the lead compensator, therefore an additional fuzzy predictive functional controller was added in the cascade. To design the outer loop fuzzy predictive controller, the fuzzy model of the inner loop had to be obtained. Due to the unknown values of R, L, c and nonlinear nature of the observed process, a fuzzy identification was used to obtain the process model.

3. Fuzzy Identification

Fuzzy modelling became a very important area of research recently. Fuzzy models are, like artificial neural networks, universal approximators. Originally, fuzzy model represents a static nonlinear function of input and output variables. The dynamical behaviour is obtained with feeding in tap-delayed input variables and feeding back tap-delayed output variables.

Generally, there are two types of fuzzy models: Takagi–Sugeno (TS) and Mamdani. TS model can be written as follows:

$$\mathbf{R}^j\colon\ \textbf{if}\ x_1\ \textbf{is}\ A_1^j\ \textbf{and}\ldots\textbf{and}\ x_N\ \textbf{is}\ A_N^j\ \textbf{then}\ y = f^j(x_1,\ldots,x_N), \tag{9}$$

where x_i are inputs, A_i^j are subsets of the input space, y is the output and f^j is a function, generally nonlinear.

Second type of fuzzy model is the Mamdani model:

$$\mathbf{R}^j\colon\ \textbf{if}\ x_1\ \textbf{is}\ A_1^j\ \textbf{and}\ldots\textbf{and}\ x_N\ \textbf{is}\ A_N^j\ \textbf{then}\ y\textbf{is}\ B^j, \tag{10}$$

where B^j are consequent fuzzy sets.

The difference between the TS and the Mamdani type lies in the consequent site of if–then rules. TS type anticipates the function for calculating the output, while Mamdani type classify output as consequent fuzzy sets. The function consequent part of TS model represents the linear model, so the rules are just for switching among different linear models. Therefore, the TS type of model needs less rules than Mamdani type for the same function.

In case of the magnetic suspension, a minor modification of the TS rule was made. The antecedent variable was not the part of the regressor. As will be seen from Section 5, the system can be satisfactory modelled as discrete third-order model without zeroes. For this reason the derivation of the fuzzy identification and the control law will be based on third-order model. The ith rule can be written:

$$\begin{aligned}&\mathrm{R}^i\colon\ \text{if } av \text{ is } A^i \text{ then}\\ &y_m(k+1) = a_{1i}y_m(k) + a_{2i}y_m(k-1) + a_{3i}y_m(k-2) + b_iu(k-D) + r_i,\end{aligned} \tag{11}$$

where $y_m(k+1)$ is the output and $y_m(k)$, $y_m(k-1)$, $y_m(k-2)$, $u(k-D)$ are the inputs of the fuzzy model. D stands for the dead time expressed by the number of samples, A^i are antecedent fuzzy sets and av is the antecedent variable, which will be described later in the paper.

Using Fuzzy mean defuzzification method, the output is expressed by the following equation:

$$\begin{aligned}y_m(k+1) &= \sum_{i=1}^{K}\beta_i(k)(a_{1i}y_m(k) + a_{2i}y_m(k-1) + \\ &\quad + a_{3i}y_m(k-2) + b_iu(k-D) + r_i).\end{aligned} \tag{12}$$

K stands for the number of rules and $\beta_i(k)$ is the normalized degree of fulfilment of ith rule at kth step. The measured process output and input is the base for parameter estimation so the y_p should be used instead of y_m for the purpose of identification. Equation (12) can be written with K equations as follows (Škrjanc and Matko, 2000):

$$\begin{aligned}
\beta_1(k)y_p(k+1) &= \beta_1(k)a_{11}y_p(k)+\beta_1(k)a_{21}y_p(k-1)+\beta_1(k)a_{31}y_p(k-2)+\beta_1(k)b_1u(k-D)+\beta_1(k)r_1\\
&\vdots\\
\beta_i(k)y_p(k+1) &= \beta_i(k)a_{1i}y_p(k)+\beta_i(k)a_{2i}y_p(k-1)+\beta_i(k)a_{3i}y_p(k-2)+\beta_i(k)b_iu(k-D)+\beta_i(k)r_i\\
&\vdots\\
\beta_K(k)y_p(k+1) &= \beta_K(k)a_{1K}y_p(k)+\beta_K(k)a_{2K}y_p(k-1)+\beta_K(k)a_{3K}y_p(k-2)+\beta_K(k)b_Ku(k-D)+\beta_K(k)r_K.
\end{aligned} \tag{13}$$

To determine parameters a_{1i}, a_{2i}, a_{3i}, b_i and r_i of the ith rule, the regression matrix $\boldsymbol{\Psi}_i$ and the output data vector $\mathbf{Y}_p^i$ should be obtained as presented in the following equations

$$\boldsymbol{\psi}_i(k) = \left[\beta_i(k)y_p(k) \quad \beta_i(k)y_p(k-1) \quad \beta_i(k)y_p(k-2) \quad \beta_i(k)u(k-D) \quad \beta_i(k)1\right], \tag{14}$$

$$\boldsymbol{\Psi}_i = \begin{bmatrix} \boldsymbol{\psi}_i(D) \\ \vdots \\ \boldsymbol{\psi}_i(k) \\ \vdots \\ \boldsymbol{\psi}_i(N-1) \end{bmatrix}, \tag{15}$$

$$\mathbf{Y}_p^i = \begin{bmatrix} \beta_i(D)y_p(D+1) \\ \vdots \\ \beta_i(k)y_p(k+1) \\ \vdots \\ \beta_i(N-1)y_p(N) \end{bmatrix}. \tag{16}$$

The vector of parameters of ith rule $\boldsymbol{\theta}_i$ is obtained by using the least squares method

$$\boldsymbol{\theta}_i = (\boldsymbol{\Psi}_i^{\mathrm{T}}\boldsymbol{\Psi}_i)^{-1}\boldsymbol{\Psi}_i^{\mathrm{T}}\mathbf{Y}_p^i, \tag{17}$$

where the elements of $\boldsymbol{\theta}_i$ are a_{1i}, a_{2i}, a_{3i}, b_i and r_i.

$$\boldsymbol{\theta}_i^{\mathrm{T}} = \left[\ a_{1i} \quad a_{2i} \quad a_{3i} \quad b_i \quad r_i\ \right]. \tag{18}$$

The steps from Equations (14), (15), (16) and (17) should be repeated for all rules. Vectors $\boldsymbol{\theta}_i$ can be joined to a matrix

$$\boldsymbol{\Theta} = \left[\ \boldsymbol{\theta}_1 \quad \boldsymbol{\theta}_2 \quad \cdots \quad \boldsymbol{\theta}_K\ \right], \tag{19}$$

where ith column represents the parameter vector of ith rule. The fuzzy model of Equation (12) can be written in the following form also called global linear model

$$\begin{aligned} y_m(k+1) &= \tilde{a}_1(k)y_m(k) + \tilde{a}_2(k)y_m(k-1) + \tilde{a}_3(k)y_m(k-2) + \\ &\quad + \tilde{b}(k)u(k-D) + \tilde{r}(k), \end{aligned} \tag{20}$$

where the parameters are

$$\begin{aligned}
\tilde{a}_1(k) &= \sum_{i=1}^{K} \beta_i(k)\boldsymbol{\Theta}_{1i},\\
\tilde{a}_2(k) &= \sum_{i=1}^{K} \beta_i(k)\boldsymbol{\Theta}_{2i},\\
\tilde{a}_3(k) &= \sum_{i=1}^{K} \beta_i(k)\boldsymbol{\Theta}_{3i},\\
\tilde{b}(k) &= \sum_{i=1}^{K} \beta_i(k)\boldsymbol{\Theta}_{4i},\\
\tilde{r}(k) &= \sum_{i=1}^{K} \beta_i(k)\boldsymbol{\Theta}_{5i}.
\end{aligned} \tag{21}$$

4. Fuzzy Predictive Functional Control

The basic idea of model based predictive control is to predict the future behaviour of the process over a certain horizon using the dynamic model and obtaining the control actions to minimize a certain criterion, generally

$$J(u,k) = \sum_{j=N_1}^{N_2} (y_m(k+j) - y_r(k+j))^2 + \lambda \sum_{j=1}^{N_u} u^2(k+j). \tag{22}$$

Signals $y_m(k+j)$, $y_r(k+j)$, $u(k+j)$ are j-step ahead predictions of the process model output, the reference trajectory and the control signal respectively. Parameter λ is the weight of the control signal energy. N_1, N_2 and N_u are minimum, maximum and control horizon respectively.

Predictive functional control (PFC) is one of MBPC method. The criterion of Equation (22) is not minimized using the time-consuming minimisation functions, but is implied in the control law, so PFC is a very appropriate method for short sampling time processes. In combination with fuzzy model is called Fuzzy PFC (FPFC).

The predictive functional controller is designed in the time domain. For the purpose of H step-ahead prediction, the model from Equation (20) should be transformed into a more compact form, for example in the state space domain

$$\mathbf{x}_m(k+1) = \tilde{\mathbf{A}}_m\mathbf{x}_m(k) + \tilde{\mathbf{B}}_m u(k) + \tilde{\mathbf{R}}_m, \tag{23}$$

$$y_m(k) = \tilde{\mathbf{C}}_m\mathbf{x}_m(k). \tag{24}$$

If the state vector $\mathbf{x}_m(k)$ is

$$\mathbf{x}_m(k) = \begin{bmatrix} y_m(k) \\ y_m(k-1) \\ y_m(k-2) \end{bmatrix}, \tag{25}$$

then matrices $\tilde{\mathbf{A}}_m$, $\tilde{\mathbf{B}}_m$, $\tilde{\mathbf{R}}_m$ and $\tilde{\mathbf{C}}_m$, become

$$\tilde{\mathbf{A}}_m = \begin{bmatrix} \tilde{a}_1 & \tilde{a}_2 & \tilde{a}_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \tag{26}$$

$$\tilde{\mathbf{B}}_m = \begin{bmatrix} \tilde{b} \\ 0 \\ 0 \end{bmatrix}, \tag{27}$$

$$\tilde{\mathbf{R}}_m = \begin{bmatrix} \tilde{r} \\ 0 \\ 0 \end{bmatrix}, \tag{28}$$

$$\tilde{\mathbf{C}}_m = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}. \tag{29}$$

Presuming

$$u(k) = u(k+1) = \cdots = u(k+H-1), \tag{30}$$

the H step-ahead prediction can be written

$$y_m(k+H) = \tilde{\mathbf{C}}_m\big(\tilde{\mathbf{A}}_m^H \mathbf{x}_m(k) + (\tilde{\mathbf{A}}_m^{H-1} + \cdots + \tilde{\mathbf{A}}_m + \mathbf{I})(\tilde{\mathbf{B}}_m u(k) + \tilde{\mathbf{R}}_m)\big). \tag{31}$$

The sum of powered $\tilde{\mathbf{A}}_m$ matrices can be simplified as

$$\tilde{\mathbf{A}}_m^{H-1} + \cdots + \tilde{\mathbf{A}}_m + \mathbf{I} = (\tilde{\mathbf{A}}_m^H - \mathbf{I})(\tilde{\mathbf{A}}_m - \mathbf{I})^{-1}. \tag{32}$$

The closed-loop response should be similar to the reference trajectory, which is the output of the reference model.

$$\mathbf{x}_r(k+1) = \mathbf{A}_r \mathbf{x}_r(k) + \mathbf{B}_r w(k), \tag{33}$$

$$y_r(k) = \mathbf{C}_r \mathbf{x}_r(k). \tag{34}$$

Matrices $\mathbf{A}_r$, $\mathbf{B}_r$ and $\mathbf{C}_r$ have to be chosen to fulfil the equation.

$$\mathbf{C}_r(\mathbf{I} - \mathbf{A}_r)^{-1}\mathbf{B}_r = 1. \tag{35}$$

In the same manner, as with the process model, the H step-ahead prediction of the reference model can be written as

$$y_r(k+H) = \mathbf{C}_r\big(\mathbf{A}_r^H \mathbf{x}_r(k) + (\mathbf{A}_r^H - \mathbf{I})(\mathbf{A}_r - \mathbf{I})^{-1}\mathbf{B}_r w(k)\big). \tag{36}$$

The main goal of FPFC is to equalize the process objective increment Δ_p and the model objective increment Δ_m at a certain horizon H. The control action is then calculated from required model increment. The process objective increment is the difference between H-step ahead predicted reference trajectory and the present process output

$$\Delta_p = y_r(k+H) - y_p(k), \tag{37}$$

$$\Delta_p = \mathbf{C}_r\big(\mathbf{A}_r^H \mathbf{x}_r(k) + (\mathbf{A}_r^H - \mathbf{I})(\mathbf{A}_r - \mathbf{I})^{-1}\mathbf{B}_r w(k)\big) - y_p(k). \tag{38}$$

The model output increment is defined as

$$\Delta_m = y_m(k+H) - y_m(k), \tag{39}$$

$$\Delta_m = \tilde{\mathbf{C}}_m\big(\tilde{\mathbf{A}}_m^H \mathbf{x}_m(k) + (\tilde{\mathbf{A}}_m^H - \mathbf{I})(\tilde{\mathbf{A}}_m - \mathbf{I})^{-1}(\tilde{\mathbf{B}}_m u(k) + \tilde{\mathbf{R}}_m)\big) - y_m(k). \tag{40}$$

As mentioned above the control action is obtained by equalizing

$$\Delta_p = \Delta_m. \tag{41}$$

The control law can be explicitly expressed by deriving the control variable $u(k)$.

$$u(k) = \frac{\mathbf{C}_r\mathbf{A}_r^H\mathbf{x}_r(k) + \mathbf{C}_r(\mathbf{A}_r^H - \mathbf{I})(\mathbf{A}_r - \mathbf{I})^{-1}\mathbf{B}_r w(k) - y_p(k) - \tilde{\mathbf{C}}_m\tilde{\mathbf{A}}_m^H\mathbf{x}_m(k) - \tilde{\mathbf{C}}_m(\tilde{\mathbf{A}}_m^H - \mathbf{I})(\tilde{\mathbf{A}}_m - \mathbf{I})^{-1}\tilde{\mathbf{R}}_m + y_m(k)}{\tilde{\mathbf{C}}_m(\tilde{\mathbf{A}}_m^H - \mathbf{I})(\tilde{\mathbf{A}}_m - \mathbf{I})^{-1}\tilde{\mathbf{B}}_m}. \tag{42}$$

The minimisation criterion of Equation (22) is on first look not implied in the proposed control law, but some way it is. Control action is calculated in the way to minimize the difference between reference model output and process model output. That can be seen from Equations (37), (39) and (41). The “saving energy” part can be considered as presumption of Equation (30). The prediction horizon H has an analogy with the parameter λ. Large H gives more time to the controller, what results in more smooth time diagram of control variable and small H makes controller more nervous. Changing the parameter λ results in the same way.

5. Real-Time Experiment

The theory of FPFC has been applied to the magnetic suspension previously stabilized with the lead compensator. The electrical part of the system from Equation (3) has very fast dynamics so the poles from Equation (1) are dominant. We can presume that the magnetic suspension is originally a second order system. The compensator adds another pole into the closed-loop system, which results in the closed-loop structure of the third order. Nevertheless, the structure of fuzzy model was established with experimenting on the identification data. The best identification result was achieved the structure was following.

TS model has antecedent fuzzy sets A_1 and A_2. Membership functions of antecedent fuzzy sets are shown in Figure 2. They have been chosen in accordance with different process dynamics when process output is increasing or decreasing. For the antecedent variable filtered derivative was chosen in the form $y(k) - y(k-10)$. Different dynamics can be seen from series of step response shown in Figure 3.

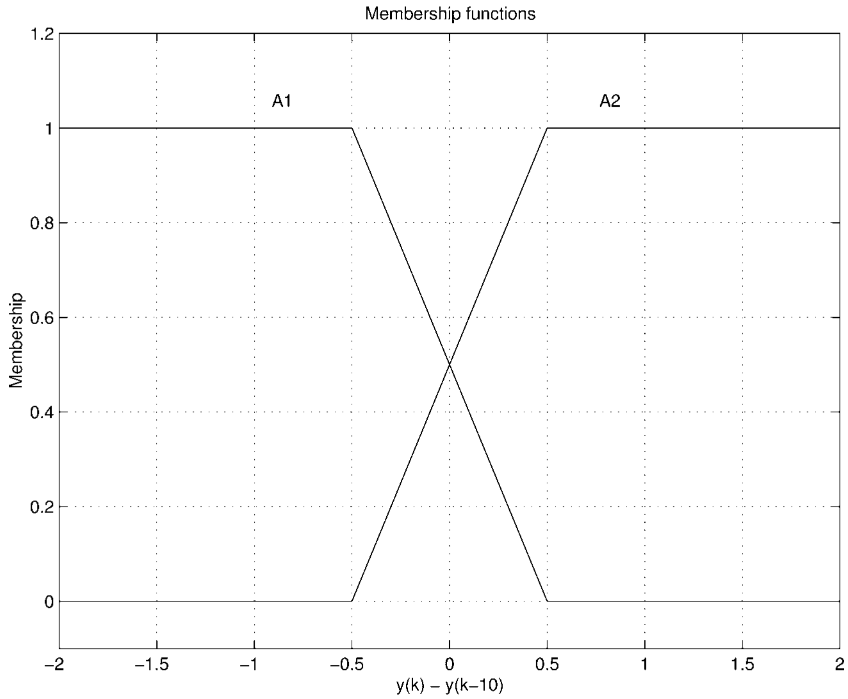

Figure 2. Antecedent fuzzy sets.

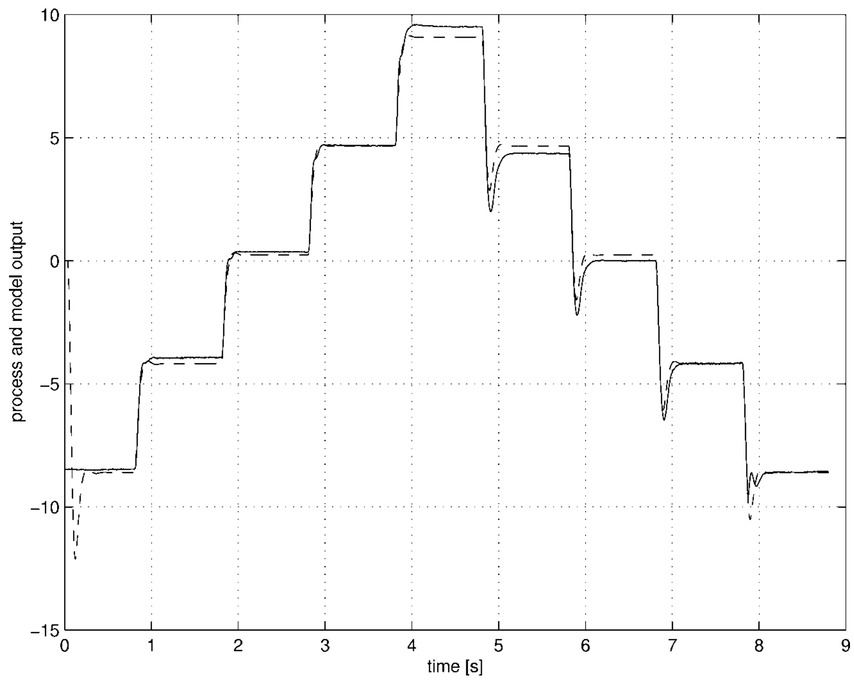

Figure 3. Validation of the model.

Experiments with the identification data sampled on the magnetic suspension device confirmed that the third order submodels without discrete zeroes are sufficient to describe the process. The identified TS model with the sampling time of 2 ms is

$$\begin{aligned}
&\mathrm{R}^1\text{: if } y(k) - y(k-10) \text{ is } A_1 \text{ then}\\
&y(k+1) = 1.3449y(k) + 0.2288y(k-1) - 0.5822y(k-2) +\\
&\qquad\qquad + 0.0188u(k-2) + 0.0199,\\
&\mathrm{R}^2\text{: if } y(k) - y(k-10) \text{ is } A_2 \text{ then}\\
&y(k+1) = 1.3174y(k) + 0.1495y(k-1) - 0.4755y(k-2) +\\
&\qquad\qquad + 0.0189u(k-2) + 0.0219.
\end{aligned} \tag{43}$$

A tight fit of the simulated output (dashed) to the real response (solid) is shown in Figure 3.

Tuning the FPFC in the phase when the process model is obtained means choosing the reference model and the prediction horizon. With the reference model, the time constant of closed loop system is determined. The third-order reference model was chosen with the respect to the model dynamics. The reference model should not be to fast if we want to avoid the oscillations, but should be fast enough to fulfil the control requirements. Three discrete poles of the reference model were placed on the same position:

$$p_1 = p_2 = p_3 = 0.94. \tag{44}$$

The prediction horizon is normally chosen to fulfil the term

$$N \leqslant H \leqslant \frac{T_r}{2T_s}, \tag{45}$$

where N is the process order, T_r is the time constant of reference model and T_s is the sampling time. In our case prediction horizon was chosen as $H = 12$. Smaller prediction horizon on one hand results in better accuracy of the predicted process output, but on the other hand, the FPFC strongly amplifies the noise of the measured process output.

Time-consuming calculating of $u(k)$ can be a problem in the case of very fast process dynamics. In each sample time the computer is dealing with different matrices $\tilde{\mathbf{A}}_m$, $\tilde{\mathbf{B}}_m$, $\tilde{\mathbf{R}}_m$ and $\tilde{\mathbf{C}}_m$. The problem was solved in the following way: For both consequent linear models of Equation (43) linear PFC-s were designed. Matrices $\tilde{\mathbf{A}}_m$, $\tilde{\mathbf{B}}_m$, $\tilde{\mathbf{R}}_m$ and $\tilde{\mathbf{C}}_m$ are composed from both consequent linear models. Since the multiplication factors of all signals of control law presented in Equation (42) are constant, can be computed in advance and represent two linear PFC-s. Linear combination was made using the membership degrees β_1 and β_2. The complex matrix computations were transformed to a simple equation

$$u(k) = \beta_1 u_1(k) + \beta_2 u_2(k), \tag{46}$$

where $u_1(k)$ and $u_2(k)$ stand for output of both linear PFC.

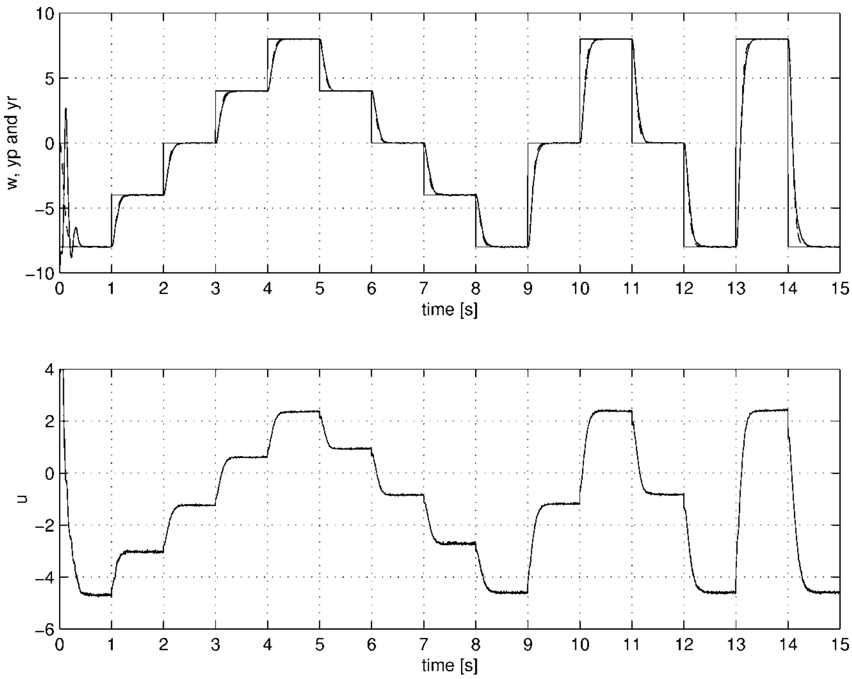

Figure 4. Response using FPFC.

In Figure 4 the real-time response of the process (solid) and the reference trajectory (dashed) are shown. The detail is presented in Figure 5.

5.1. COMPARISON OF FPFC WITH PID AND PFC

Finally, the comparison with a PID control and a linear PFC was made. The Smith-predictor was applied in the PID controller case, to assure equal conditions for all controllers considering the dead-time. The PID controller was modified so the differential input was connected direct to the process output. The structure of the PID controller can be seen from the equation

$$U(z^{-1}) = K_p E(z^{-1}) + K_i T_s \frac{1}{1-z^{-1}} E(z^{-1}) + \frac{K_d}{T_s}(1-z^{-1}) Y_p(z^{-1}). \quad (47)$$

The parameters of the PID controller were obtained with optimisation using the ITAE criterion. The goal was good response to the positive step from -3 to 3 V. Resulted parameters were $K_p = 0.185$, $K_i = 3.70$ and $K_d = 0.005$, and the sampling time was $T_s = 2$ ms.

The PFC design was based on the linear model of the process obtained from the fuzzy model. The linear model parameters are mean values of both consequent linear models of the TS model. Obtaining the linear model can be also explained, as setting the antecedent variable $av = y_p(k) - y_p(k-10)$ to zero. That gives us

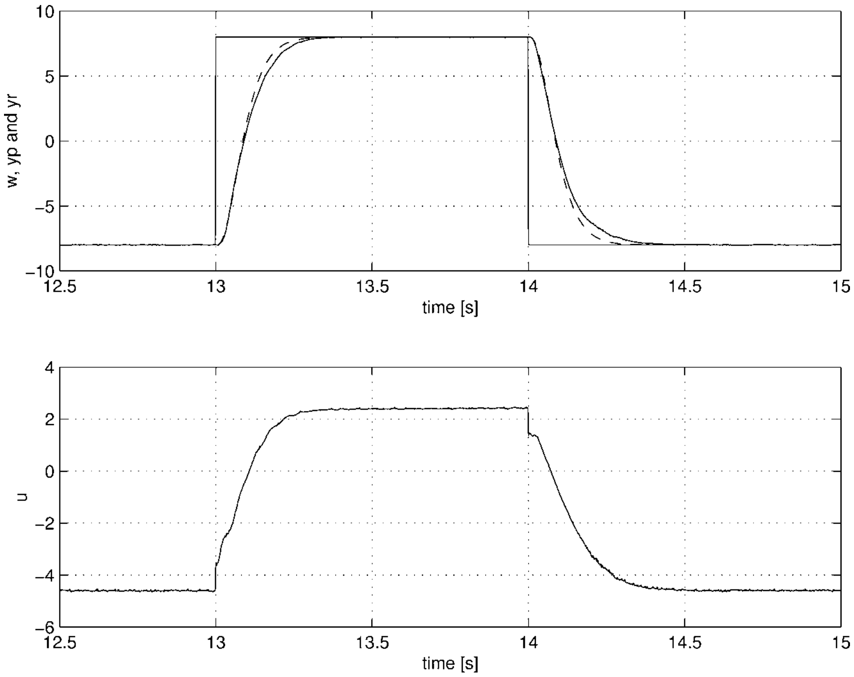

Figure 5. Detail of Figure 4.

the linear model, which step response is something between step responses of both consequent models. The linear model is

$$y_p(k+1) = 1.3312y(k) + 0.1891y(k-1) - 0.5289y(k-2) + \\ + 0.0188u(k-2) + 0.0209. \quad (48)$$

The PFC design is similar to the FPFC design described in Equation (23) to Equation (42), but based on linear model.

Comparison of the control techniques in real-time on the pilot plant can be seen from Figure 6 and detailed look is best shown in Figure 7. The FPFC response is plotted with the dash-dot, the linear PFC response with the dashed and the PID response with the solid line.

Combining all three responses together, the following can be said: When comparing the PID control and the linear PFC, the difference lies in the modern prediction law, and when comparing the PFC with the FPFC, the difference is in the nonlinear structure of the FPFC. The difference between the PID and the FPFC is as well in the modern prediction law as in the nonlinear structure.

The responses using the FPFC and the PFC are not very different. The difference between both controllers is in the case of fast decreasing of the process output. Considering the nonlinearity as the linear model parameters mismatch, this can be another proof of the robustness of the predictive functional control.

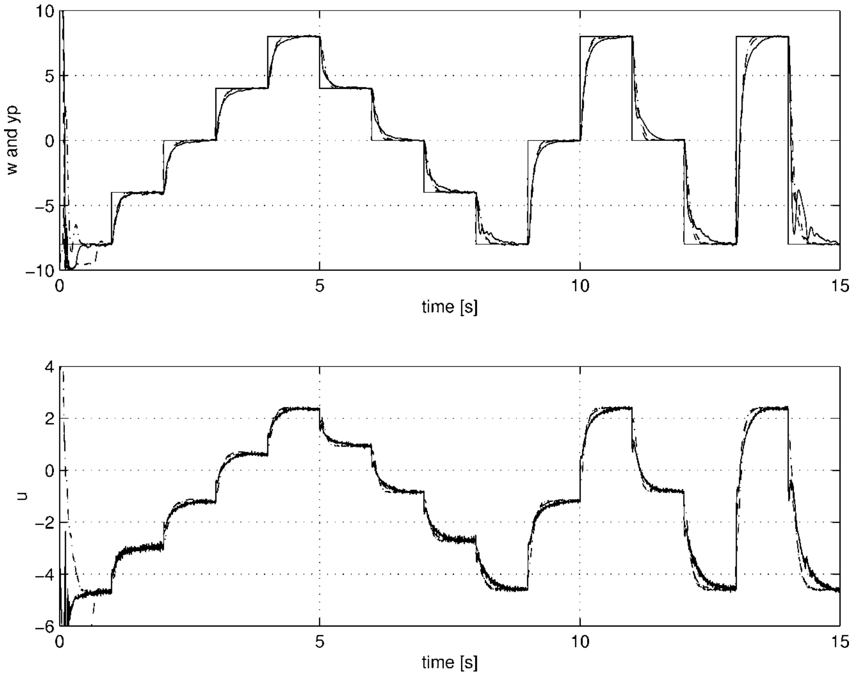

Figure 6. Comparison of step responses using different controllers.

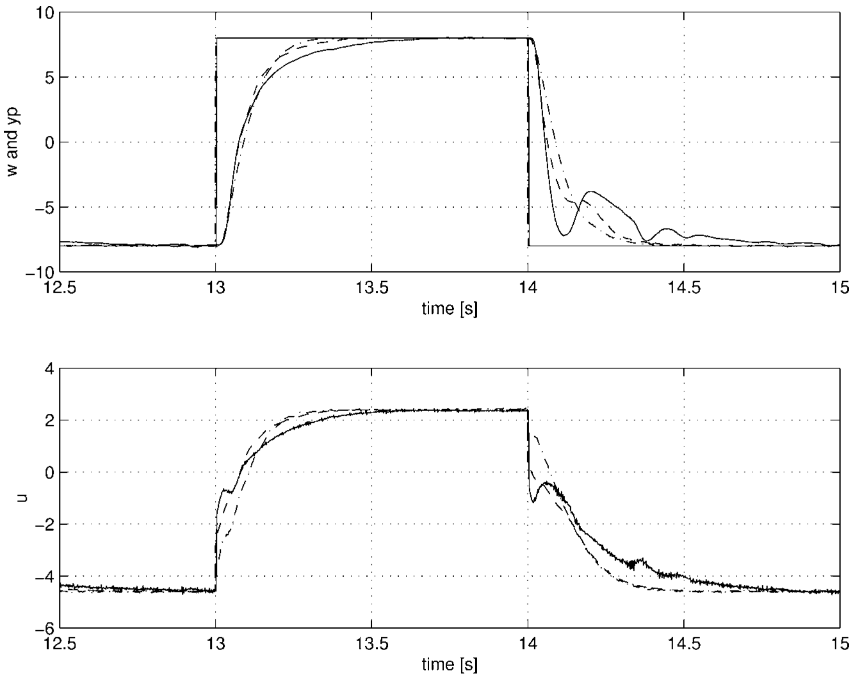

Figure 7. Detail of Figure 6.

Comparing with the PID control, the FPFC is better in all cases. The settling time on positive changes of reference using the PID control is twice longer, than using the FPFC. Another drawback of the PID control was appearance of oscillations in the lower part of the operating range.

6. Conclusion

The nonlinear prediction control law is presented in the paper. The theory of FPFC is implemented to the nonlinear process with very fast dynamics. As seen from the comparison with conventional PID control, the FPFC exhibits very good performance. The nonlinear structure of the FPFC offers some extra advantage, which can be seen from the comparisons with the linear PFC. Its great robustness in the presence of model inaccuracies and unmodeled dynamics certified with real-time experiment makes FPFC convenient for a great number of applications. Regarding to the idea of magnetic bearing, the FPFC promises high-quality control.

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